A CHARACTERIZATION OF METRIZABILITY THROUGH GAMES

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Metrizable spaces are one of the most important concepts in mathematics. For this reason, the quest for characterizing metrizability has always risen a lot of interest in general topology, leading to results as Urysohn metrization theorem, Nagata-Smirnov metrization theorem, Arhangel'skij metrization theorem, ...

Metrizability has a natural generalization that occurs when the usual range of the metric $\mathbb{R}$ is replaced by another structure $\mathbb{G} = \langle G, +_{\mathbb{G}}, 0_{\mathbb{G}}, \leq_{\mathbb{G}} \rangle$. We call $\mathbb{G}$-metric a space X with a function $d : X \times X \to \mathbb{G}$ where the usual rules of the metric are stated with $\mathbb{R}$ replaced by $\mathbb{G}$. This approach is very inclusive: for example, an ultrametric can be seen as a $\mathbb{G}$-metric with values in the monoid $\mathbb{G} = \langle \mathbb{R}, \max, 0, \leq \rangle$ where the usual real operation is replaced by max.

In this talk, I will first do a brief survey on metrization theorems, presenting some of the classical ones and their analogues for $\mathbb{G}$-metrics. Then, I will present a (hopefully) new characterization of both metrizability and μ-metrizability in terms of topological games. These results are part of a joint work with Luca Motto Ros.

References

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